ARPA Network Information Center
Stanford Research Institute
Menlo Park, California 94025

NIC # 16750
Packet Radio Temporary Note # 58

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31 May 1973

A COMPARISON OF SPREAD SPECTRUM WITH ALOHA FOR PACKET RADIO NETWORKS

ABSTRACT

This note describes a model for one spread spectrum modulation scheme. The model is used to derive the channel throughput, comparisons with other schemes for packet transmission under identical conditions are made. The analysis is then extended to coded systems and a bound on the average probability of error is found. Other considerations such as privacy, error protection, feedback path are briefly mentioned.

INTRODUCTION

In this note, a model is described for a spread spectrum type of digital modulation. This model is used to calculate the channel throughput in terms of bandwidth. The analysis assumes there is no external noise. This type of analysis enables comparison of spread spectrum with other types of packet transmission schemes with the same bandwidth such as ALOHA and TDM. It is shown that the throughput of spread spectrum modulation is not as good as that of ALOHA type modulation.

The analysis is further extended to include encoded spread spectrum systems with binary signalling. The results show that the total rate is a function of the bandwidth and error exponent in limiting cases.

There has been recent interest in the application of spread spectrum to digital communications. Gold [1] has shown methods to use spread spectrum for TDM. Kaiser [2] discussed the application of spread spectrum for packet radio networks.

SPREAD SPECTRUM MODEL

The spread spectrum digital modulation scheme uses classical matched filters or correlation detectors. In the usual matched filter type of detection, the receiver must know all transmitted wave forms exactly. The receiver correlates the received wave form with the set of possible wave forms and excludes all but one. The merit of such a receiver is determined by the degree of

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selection and degree of exclusion. The selection process is determined by the exactness of matched filter realization and the exclusion of wave forms is controlled by the orthogonality, or near orthogonality, of the wave forms. Spread spectrum as the name implies spreads the spectrum of the signal being conveyed over a wider bandwidth than would be required by more conventional modulation techniques.

TRANSMITTER

Each user has one transmitter and a receiver. Each user's transmitter is allotted a unique signal, $S_\ell(t)$ (which is like a carrier signal) called the 'signature signal'. This signal is represented in terms of a set of 2TW orthonormal functions $\{\phi_j(t): j = 1,2,\ldots 2TW\}$ and the ℓth user's binary signature sequence $S(\ell) = \{S_1(\ell), S_2(\ell)\ldots,S_{2TW}(\ell)\}$ as

$$S_\ell(t) = \sum_{j=1}^{2TW} (-1)^{S_j(\ell)} \frac{1}{\sqrt{2TW}} \phi_j(t) \qquad (1)$$

where T is the duration of signature signal, W is the bandwidth of signature signal, the elements of the ℓth user's signature sequence, $S_j(\ell)$, are binary and the user index $\ell \in \{1,2,\ldots L\}$.

Each user sends binary digits by amplitude modulating with -1 or +1 his signature signal.

CHANNEL

The received signal Y(t) is the sum of all the transmitted signals:

$$Y(t) = \sum_{k=-\infty}^{\infty} (-1)^{b_k} S_{\ell_k}(t-\tau_k) \qquad (2)$$

where user $\ell_k \in \{1,2,\ldots L\}$ transmits the binary information digit $b_k \in \{0,1\}$ at time τ_k.

RECEIVER

The receiver for user ℓ is essentially a filter matched to the signature signal, $S_\ell(t)$, followed by a decision device as shown in Fig. 1.

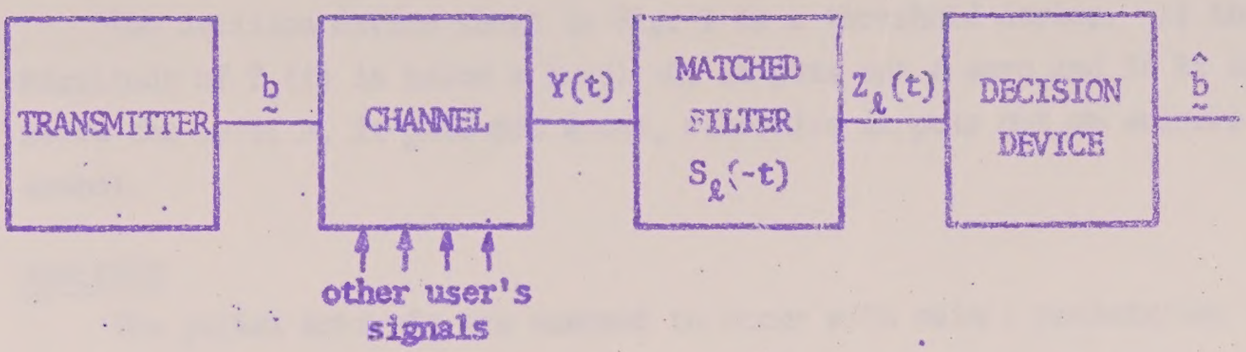


Fig. 1 Spread Spectrum Model for the ℓth User

Thus the output of the ℓth matched filter $Z_\ell(t)$ can be written as:

$$Z_\ell(t) = Y(t) * S_\ell(-t)$$

$$= \sum_{k=-\infty}^{\infty} (-1)^{b_k} S_{\ell k}(t-\tau_k) * S_\ell(-t)$$

$$= \sum_{k=-\infty}^{\infty} (-1)^{b_k} C_{\ell k,\ell}(t-\tau_k) \qquad (3)$$

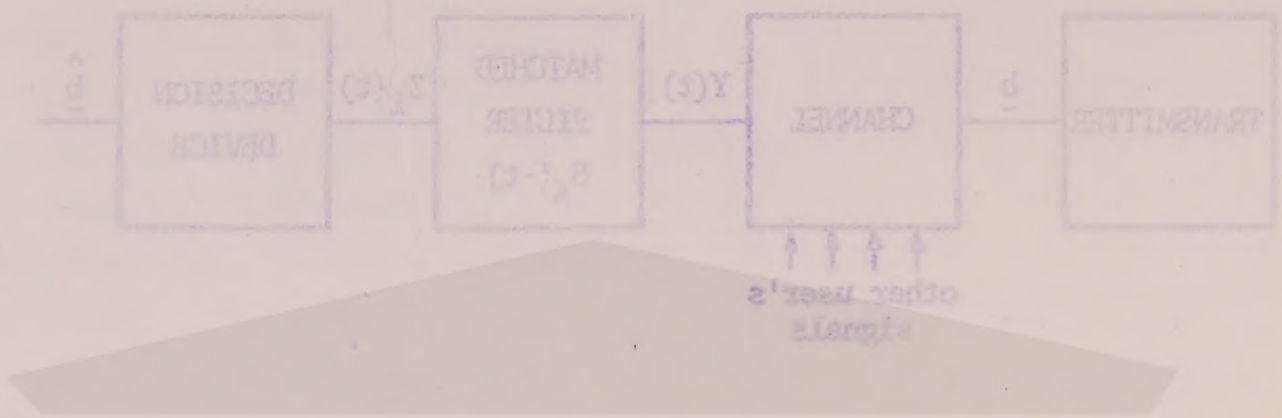
where the cross correlation between the ℓth and ℓ_kth users signature signals is

$$C_{\ell_1 \ell_k}(\tau) = \int S_\ell(\alpha)\, S_{\ell k}(\alpha+\tau)\, d\alpha \ .$$

Thus when ℓ_0 is ℓ, the output of the matched filter is

$$Z_\ell(t) = (-1)^{b_0} C_{\ell,\ell}(t-\tau_0) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} (-1)^{b_k} C_{\ell,\ell_k}(t-\tau_k) \qquad (4)$$

Even though k varies from $-\infty$ to ∞ in the summation, the only terms which will contribute to $Z_\ell(t)$ at any time t are those with emission times such that $|t-t_k| < T$.



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The decision device shown in Fig. 1 is a threshold device. If the magnitude of $Z_\ell(t)$ is below a level -A, it puts out a zero and if it is above the level A, it puts out an erasure symbol.

ANALYSIS

The packet arrivals are assumed to occur with rate λ packets/sec in the following analysis. Also, it is assumed that there is no other noise.

Consider a typical term in the summation (4), $C_{\ell,\ell'}(t)$ which is the cross correlation between the ℓth and the ℓ'th signature signals.

$$C_{\ell,\ell'}(t) = \int_{-\infty}^{\infty} S_\ell(\alpha)\, S_{\ell'}(\alpha - t)\, d\alpha \quad .$$

By using (1), $S_\ell(t)$ and $S_{\ell'}(+t)$ can be written in terms of the orthonormal functions.

$$C_{\ell,\ell'}(t) = \sum_j \sum_k \frac{(-1)^{S_j(\ell)+S_j(\ell')}}{2TW} \int_{-\infty}^{\infty} \phi_j(\alpha)\phi_k(\alpha+t)\, d\alpha \quad . \qquad (5)$$

Now if the orthonormal basis consists of time shifted pulses of duration 2W

$$\phi_j(t) = S\left(t - \frac{j}{2W}\right) \qquad (6)$$

as shown in Fig. 2,

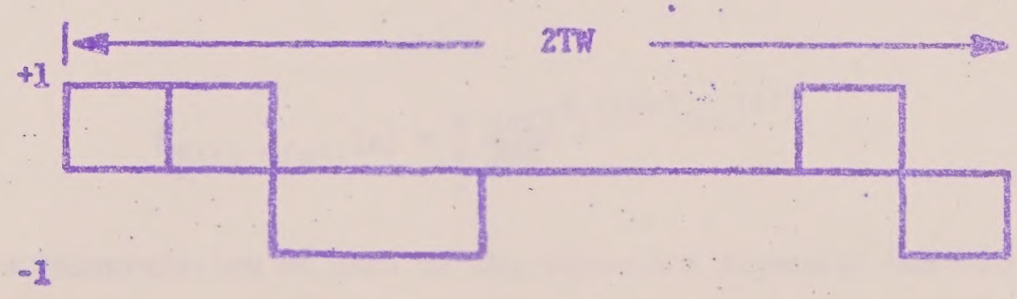


Fig. 2 T Pulses of 2W Duration With Values +1 or -1

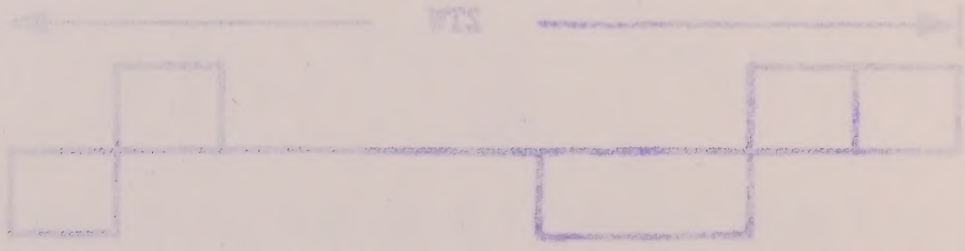


Fig. 1 T Pulses of TW Duration With Values +1 or -1

Without loss of generality we assume

$$t = \frac{n}{2W} + \tau \quad \text{where} \quad |\tau| = \frac{1}{W} \quad \text{and } \tau \in \{0, \frac{1}{2W}\}$$

then

$$C_{\ell,\ell'}(\frac{n}{2W}+\tau) = \sum_j \sum_k \frac{(-1)^{S_j(\ell)+S_k(\ell')}}{2IW} \int_{-\infty}^{\infty} S(\alpha - \frac{j}{2W}) S(\alpha + \frac{n-k}{2W} + \tau) d\alpha$$

$$= \sum_j \sum_k \frac{(-1)^{S_j(\ell)+S_k(\ell')}}{2IW} \int S(\gamma)\, S(\gamma + \frac{n-k+j}{2W} + \tau) d\gamma \quad .$$

Since $C_\phi(x)$ is zero $|x| > \frac{1}{2W}$, we get

$$\therefore -k+j = 0 \text{ or } -1 \quad .$$

Hence the cross correlation between the signature signals is essentially by the cross correlation between signature sequences:

$$C_{\ell,\ell'}(\frac{n}{2W}+\tau) = \sum_j \frac{(-1)^{S_j(\ell)+S_{j+n}(\ell')}}{2IW} C_S(\tau) + \sum_j (-1)^{S_j(\ell)+S_{j+n+1}(\ell')} C_S(-\frac{1}{2W}+\tau)$$

$$= C_S(\tau)\, \tilde{C}_{\underline{S}(\ell)\underline{S}(\ell')}(n) + C_S(\tau - \frac{1}{2W})\, \tilde{C}_{\underline{S}(\ell)\underline{S}(\ell')}(n+1) \qquad (7)$$

where the cross correlation between the ℓth and the ℓ'th user's signature sequences is

$$\tilde{C}_{\underline{S}(\ell),\underline{S}(\ell')}(n) = \sum_j \frac{(-1)^{S_j(\ell)+S_{j+n}(\ell')}}{2IW} \qquad (6)$$

The autocorrelation at zero of the signature sequences has been normalized to unity: by the use of the Schwartz Inequality one can show that the magnitude of the cross correlation is bounded by unity.

If the $S_j(\ell)$'s are assumed to be independent identically distributed and equally likely to be one or zero then when $n \neq 0$,

$$E\,\tilde{C}_{\underline{S}(\ell)\underline{S}(\ell')}(n) = E\left[\sum_j \sum_i (-1)^{S_j(\ell)+S_{j+n}(\ell')} (-1)^{S_i(\ell)S_{i+n}(\ell')} \frac{1}{(2TW)^2}\right]$$

$$= \sum E\left[(-1)^{S_j(\ell)}\right]^2 \cdot E\left[(-1)^{S_{j+n}(\ell)}\right]^2 \frac{1}{(2TW)^2}$$

$$= \frac{1}{(2TW)^2}\,[2TW-|n|]$$

$$= \frac{1}{2TW}\left[1 - \frac{|n|}{2TW}\right] \qquad (9)$$

which can be plotted as shown in Fig. 3.

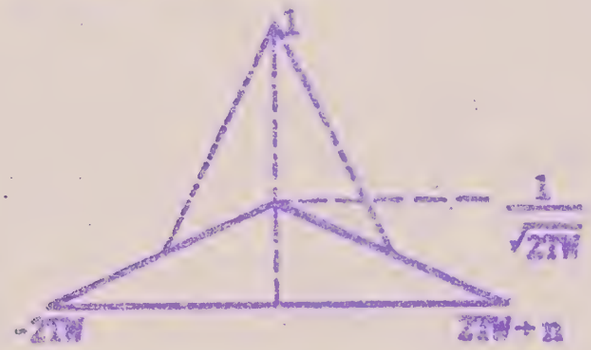


Fig. 3 The Cross Correlation Between Signature Sequences

The summation term in (4) then has a mean zero and a variance $\lambda \cdot 2T \cdot \frac{1}{2TW} \cdot \frac{1}{2} = \frac{\lambda}{2W}$. The variance expression needs some explanation. The time interval under consideration has length 2T. The average number of packets sent in this time interval is $2T\lambda$; the variance of each term is $\frac{1}{2TW}\left(1 - \frac{|n|}{2TW}\right)$. The factor $\frac{1}{2}$ arises from averaging the triangular cross correlation.

Since the cross correlation term arises from signature signals of other users and these signals are chosen to have low cross correlation properties, the variance term should be much less than A which is less than unity.

$$\frac{\lambda}{2W} << 1 \quad \text{or} \quad \lambda << 2W \qquad (10)$$

If the summation term has a large number of contributing terms, the central limit theorem can be applied. This results in summation term being Gaussian with mean zero and variance $\frac{\lambda}{2W}$. The probability of error consists of three terms:

1) the probability of deciding something was sent when nothing was sent, $P_r\{|noise|>A\}$

2) the probability of making a mistake when something was sent, $P_r\{noise<A-1\}$

The following tables gives the value of $\frac{W}{\lambda}$ for various values of error probability. The two probabilities of error mentioned above are assumed to be equal; values for each one are then determined accordingly.

P_e	$\frac{W}{\lambda}$	A
10^{-1}	2.3	1.08
10^{-2}	6.6	1.005
10^{-3}	10.5	1.002
10^{-4}	15	1.001
10^{-5}	26	1.0

A probability of error of at least 10^{-4} would be necessary in practice. Thus the bandwidth required is really many times that of an ALOHA system.

Eq. (10) shows that the maximum throughput can be expressed in terms of bandwidth; this allows one to compare spread spectrum with other schemes such as classical (unslotted) ALOHA, slotted ALOHA and time division multiplexing.

For the case of:

unslotted ALOHA

$$\text{Max throughput} = \frac{\text{Max rate}}{2e} = \frac{2W}{2e} = \frac{W}{e} \quad ; \tag{11}$$

slotted ALOHA

$$\text{Max throughput} = \frac{\text{Max rate}}{e} = \frac{2W}{e} \tag{12}$$

time division multiplexing

$$\lambda = 2W \tag{13}$$

spread spectrum

$$\lambda << 2W \quad \text{or} \quad \lambda = a \cdot 2W \quad \text{where} \quad a << 1 \tag{14}$$

Equations (11)-(14) show that

1) spread spectrum is always inferior to TDM
2) if a is between $\frac{1}{2e}$ and $\frac{1}{e}$ spread spectrum is comparable to classical ALOHA, but inferior to slotted ALOHA
3) if $a < \frac{1}{2e}$, ALOHA schemes are better than spread spectrum.

In most practical cases $a < \frac{1}{2e}$, hence spread spectrum does not perform as well as ALOHA with regard to throughput.

It can easily be seen from the above analysis that the success of spread spectrum depends on the availability of sequences for signature signals which have low cross correlation properties. Gold [1] has described types of sequences which have good periodic, not aperiodic cross correlation. In particular they have:

length of sequence = 2^n

generation: linear feedback shift register

periodic correlation

$$C_{\ell,\ell'}(t) = \begin{cases} (-1) & \ell=\ell_k \\ -2^{-n/2} \text{ or } 2^{-(n+2)/2} & \ell\neq\ell_k \end{cases} \tag{15}$$

It can also be seen that the argument was critically dependent on the cross correlation terms having variance $\frac{1}{2W}$.

At the recent communication technology workshop L. R. Welch of USC [3] gave a bound on maximum cross correlation between signature type sequences:

$$C_{\ell,\ell_k}(t) \geq \frac{C_0}{\sqrt{2W}} \tag{16}$$

where C_0 is constant between 1 and 2.

Furthermore his argument is essentially that the maximum is greater than the average over a particular ensemble.

ANALYSIS OF CODED SYSTEMS

All the above analysis was done without any encoding of data. Below the coded system is analyzed. The performance characteristics of the coded systems are bounded by using the well known techniques of union bound, Chernoff

bound and random coding [4].

The following results are applicable to communications system with additive white gaussian channels and binary signalling. The average probability of error is given by

$$P_e \leq 2^{-2TW[\frac{R}{2W} + R_0]} \tag{17}$$

where the exponential bound parameter

$$R_0 = -\log_2\left[\frac{1+e^{-\frac{P_s}{2WN_0}}}{2}\right] \tag{18}$$

where P_s is the average power in the signals, R is the rate of sending packets, 2TW is the dimensionality of the signal set, and $\frac{N_0}{2}$ is the noise variance. From (17), it can be easily seen that if $\frac{R}{2W} < R_0$ then $P_e \to 0$ as $T \to \infty$ which is the desired performance.

This channel can be pictorially represented as in Fig. 4.

Fig. 4 Additive White Gaussian Channel With Binary Signalling

The equivalent spread spectrum channel model is shown in Fig. 5 which is similar to Fig. 4. The above results can be directly applied to encoded spread spectrum models.

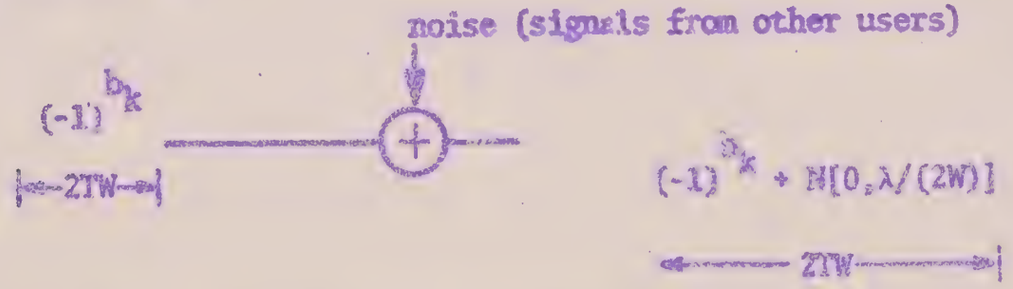


Fig. 5 Equivalent Spread Spectrum Channel Model

A comparison of noise variances gives:

$$\frac{N_0}{2} = \frac{\lambda}{2W} \quad \text{or} \quad N_0 = \frac{\lambda}{W} \quad ; \tag{19}$$

a comparison of the coefficient of $(-1)^{b_k}$ gives

$$1 = \sqrt{\frac{P_s}{2W}} \quad . \tag{20}$$

Since each user is allotted a signature signal the number of signature signals/sec is equal to rate of packets/sec.

Thus if λ = average number of signature signal/sec, R = average number of information digits per signature digit per user, L = total number of users, then R_T = average number of information digits/sec = R.L or

$$R = \frac{R_T}{\lambda L} \tag{21}$$

Substituting this value for R and using equations 14, 20, R_0 can be written as

$$R_0 = -\log_2 \left(\frac{1+e^{-W/\lambda}}{2} \right) \tag{22}$$

or

$$P_e \leq 2^{-2TW\left[-\frac{R}{2W}+R_0\right]}$$

$$= 2^{-2TW\left[-\frac{RT}{2\lambda LW} + R_0\right]} \quad . \tag{23}$$

Now introducing a new parameter S which is called the error exponent, the average probability of error can be written as:

$$P_e \leq 2^{-ST} \tag{24}$$

From (24) and (23)

$$S = -\frac{R_T}{\lambda L} + 2N \cdot R_0 \qquad (25)$$

From (25) the approximate expression for R_T can be derived in terms of L, N and S

$$R_T = \lambda L[2N R_0 - S] \qquad (26)$$

Equation (26) shows that the total rate is dependent on the number of users, the rate of transmission, the bandwidth, R_0 and S.

A limiting case of (26) can easily be derived by setting the value of λ equal to $\frac{2W}{L}$. This value of λ corresponds to the case where each user continuously begins a new signature signal at the end of each chip; thus he is sending T packets every time instant. For this hypothetical case,

$$R_0 = -\log_2 \left(\frac{1+e^{-L/2}}{2} \right) \qquad (27)$$

and

$$R_T \leq L \left[R_0 - \frac{S}{2W}\right] \qquad (28)$$

A more realistic case is $\lambda = \frac{2W}{L}$. This corresponds to each user sending packets continuously. For this case

$$R_0 = -\log_2(1+e^{-L/2T}) \qquad (29)$$

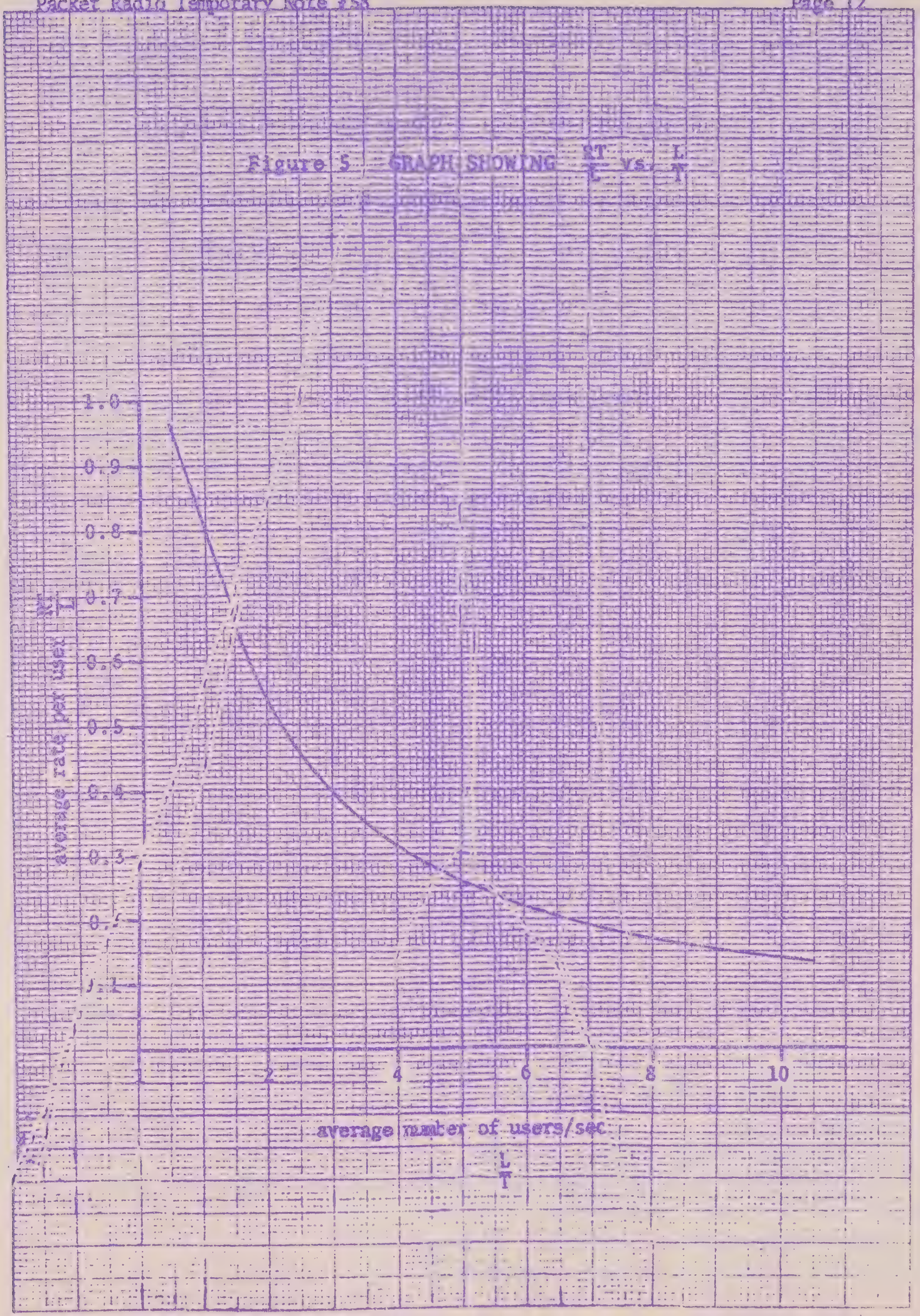
and

$$R_T \leq \frac{L}{2W} [2NR_0 - S]$$

$$= L \left[R_0 - \frac{S}{2W}\right] \qquad (30)$$

In Fig. 5, the average rate per user $\frac{R_T}{L}$ is plotted as a function of the average number of users per second, $\frac{1}{T}$ for the value of S=0. From this figure the following observations are made.

Figure 5 GRAPH SHOWING $\frac{RT}{F}$ vs. $\frac{L}{T}$



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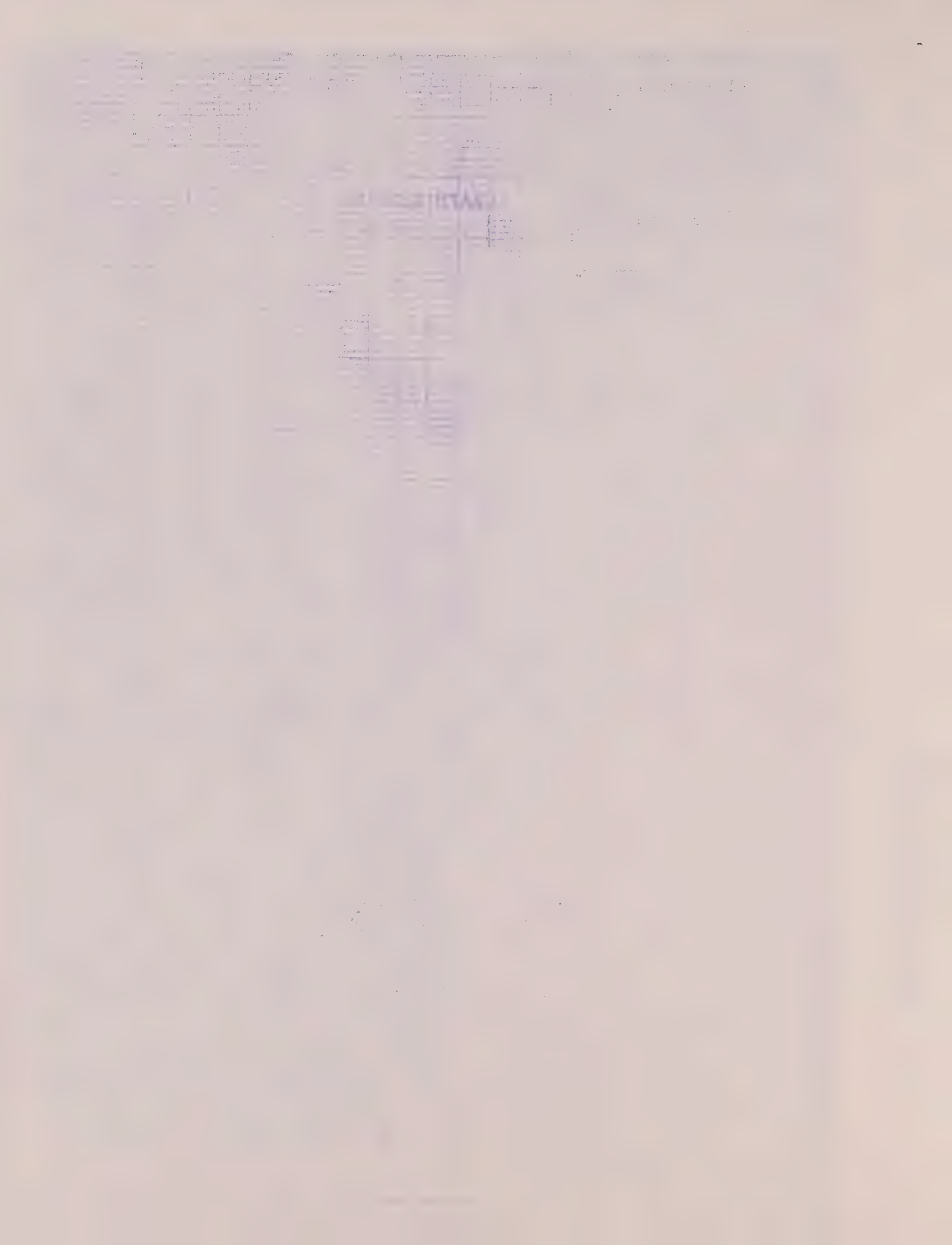

1) When $\frac{L}{T}$ is 4 we get $\frac{R_T}{L} = 0.318$ assuming S=0. But in practice S≠0, hence $\frac{R_T}{L}$ will be less than 0.318.
2) The curve represents an upper bound for $\frac{R_T}{L}$ since S is assumed to be zero.

OTHER CONSIDERATIONS

The assets of spread spectrum are large energy with a peak power constraint, privacy and the absence of feedback path. As regards error detection and error recovery spread spectrum does not offer any easy solution. Since each receiver puts out a zero or non-zero sequence there is no simple method of detecting errors once the decision is made signature signals do offer privacy.

Another problem to be considered is spread spectrum using slots.

Recently Fralick [5] published some results of throughput calculations for spread spectrum with capture. The analysis mainly considered the time capture effect, shape of autocorrelation function, blocking and criterion for interference to calculate the throughput. His results also indicate that the spread spectrum ALOHA mode is inferior to the slotted ALOHA mode with regards to throughput.

CONCLUSIONS

The above analysis shows that the spread spectrum does not do better than ALOHA as regards maximum throughput. The real advantage of spread spectrum is privacy, the absence of a feedback path and the large energy with a peak power constraint. The other disadvantage is the lack of error protection.

The bound for the total rate of transmission in an encoded system is found to be a function of bandwidth, number of users and average number of users per second. Further investigations should be done for slotted spread spectrum systems.

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[1] Gold, "Optimal Binary Sequences for Spread Spectrum Multiplexing," IEEE Transactions on Information Theory, October 1967, pp. 619-21.

[2] Kaiser, "Spread Spectrum Considerations," PR Note #19.

[3] Welch, unpublished results - at the Communications Technology Workshop, April 1973.

[4] Wozencraft & Jacobs, "Principles of Communication Engineering, Chapter 6.

[5] Fralick, "Throughput for Time Capture Spread Spectrum Packet Radio Note #43".

ACKNOWLEDGEMENT

The author is highly indebted to Dr. Gaarder for his helpful comments and suggestions at each stage.